

# Advanced Calculus- Notes

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## 1 Introduction

This is a text in production. Check back for updates. This text will compile a series of notes taken regarding various methods of analysis and in general will attempt to develop an interesting range of externalities related to the traditional Calculus sequence. Problems and proofs will be included. In addition, this text will serve as practice for LaTeX typesetting. Images will be handled in Matlab and Postscript (particularly for fractals). Topics covered will include:

- The bivalence of strong and weak induction via the well-ordering-principle of  $\mathbb{N}$
- Solution formulae for low-degree polynomials and the impossibility on the quintic
- Euler's identity, complex numbers, and the symmetry of circular and hyperbolic functions
- Elliptic functions and Elliptic integrals
- Proofs of Sandwich Theorem
- Proofs of Inverse Function Theorem
- Topics in Mean Value Theorems and Cauchy's MVT, Rolle's Theorem
- Brief Differential Geometry
- Generalized L'Hopitals Rule
- A 'Discrete' L'Hopitals- Stolz-Cesaro Theorem
- Cersaro Summation and other strange things
- Error in Standard Linear Approximation
- Some Non-Standard Analysis and Increment Theorem
- Generalizations of Pappus's Three Theorems and Parallel Axis Theorem

- Advanced Integration Techniques
- Proof of Generalized Chain Rule
- Reynolds Transport Theorem
- Jacobians
- Interlude into Differential Forms, Manifolds, and Helmholtz's Theorem
- Multivariable implicit differentiation
- Generalized Stokes Theorem
- Ostrogradsky's Method
- Newton-Rhapson
- Osborn's Rule
- Cauchy's Formula for Repeated Integration
- Continuity and Holder Conditions
- Irrotationality
- Centroid of a Solid Semiellipsoid using Generalized Pappus's Centroid Theorem
- Gamma Functions
- Stirling Approximations
- Hamiltonian's and Laplacians